Online Gradient Descent with Momentum

Dealing with Transaction Costs in Portfolio Optimization

E. Vittori, M. Bernasconi de Luca, F. Trovó, M. Restelli

IMI CORPORATE & INVESTMENT BANKING
INTESA SANPAOLO

POLITECNICO MILANO 1863

Table of contents

Online Portfolio Optimization - Context

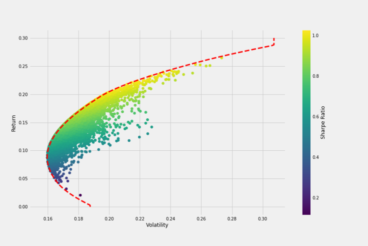
Introduction

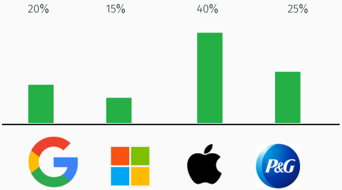
Modern Portfolio Optimization

- Statistical assumptions on the stock dynamics
- Backward looking

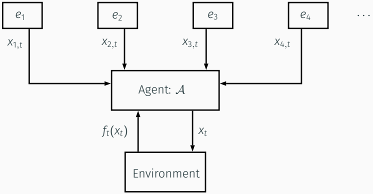
Online Portfolio Optimization

- Adversarial market
- From Expert Learning field

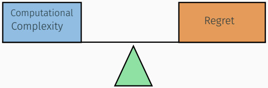




Online Learning with Experts Advice



- Regret: $R_T = \sum_{t=1}^{T} f_t(x_t) - \inf_{e \in \mathcal{E}} \sum_{t=1}^{T} f_t(x_{e,t})$
- Per-round Computational Complexity



Problem Formulation

From Expert Learning to Online Portfolio Optimization

- the experts are Constant Rebalancing Portfolios (CRP)
- $\mathbf{x}^* = \arg\inf_{x \in \Delta_{M-1}} \sum_{t=1}^{T} f_t(\mathbf{x})$ is the Best CRP
- $\mathbf{x}_t \in \Delta_{M-1}$ is the portfolio allocation
- $\mathbf{y}_t = \left(\frac{p_{t,1}}{p_{t-1,1}}, \ldots, \frac{p_{t,M}}{p_{t-1,M}} \right)$ are the returns of the stocks
- $f_t(\mathbf{x}) = -\log(\langle \mathbf{x}, \mathbf{y}_t \rangle)$ is the loss

Regret in Online Portfolio Optimization

$$R_T = \log(W_T(\mathbf{x}^*, \ldots, \mathbf{x}^*)/W_T(\mathbf{x}_1, \ldots, \mathbf{x}_T)),$$

where $W_T(\mathbf{x}_1, \ldots, \mathbf{x}_T) = \prod_{t=1}^{T} \langle \mathbf{x}_t, \mathbf{y}_t \rangle$ is the wealth

Limitations: No transaction costs

Total Regret: Adding Transaction Costs

Total Regret

$$R_T^C = \underbrace{\log(W_T(\mathbf{x}^*, \ldots, \mathbf{x}^*)/W_T(\mathbf{x}_1, \ldots, \mathbf{x}_T))}_{R_T\text{: standard regret}} + \underbrace{\gamma \sum_{t=1}^{T} ||\mathbf{x}_t - \mathbf{x}_{t-1}||_1}_{C_T\text{: transaction costs}},$$

γ is the proportional transaction rate for buying and selling stocks

Dealing with Transaction Costs - Proposed Solution

Online Gradient Descent with Momentum

Algorithm 1 OGDM in OPO with Transaction Costs

Require: learning rate sequence $\{\eta_1, \dots, \eta_T\}$, momentum parameter sequence $\{\lambda_1, \dots, \lambda_T\}$

Set $\mathbf{x}_1 \leftarrow \frac{1}{M}\mathbf{1}$
for $t \in \{1, \dots, T\}$ **do**
    Select $\mathbf{x}_{t+1} \leftarrow \Pi_{\Delta_{M-1}}\left(\mathbf{x}_t + \eta_t \frac{\mathbf{r}_t}{\langle \mathbf{r}_t, \mathbf{x}_t \rangle} - \frac{\lambda_t}{2}(\mathbf{x}_t - \mathbf{x}_{t-1})\right)$
    Observe $\mathbf{r}_{t+1}$ from the market
    Get wealth $\log(\langle \mathbf{r}_{t+1}, \mathbf{x}_{t+1} \rangle) - \gamma ||\mathbf{x}_{t+1} - \mathbf{x}_t||_1$
end for

Total Regret

$$R_T^C \leq \left[\frac{D^2}{K_\eta}\left(\frac{1}{2} + K_\lambda\right) + K_\eta \tilde{G}\left(2\gamma\sqrt{M} + \tilde{G}\right)\right]\sqrt{T}$$

where $D = \sup_{\mathbf{x},\mathbf{y} \in X} ||\mathbf{x} - \mathbf{y}||_2$, $\tilde{G} = \sup_{\mathbf{x} \in X} ||\nabla f_t(\mathbf{x})||_2 + \frac{DK_\lambda}{2K_\eta}$, $\eta_t = \frac{K_\eta}{\sqrt{t}}$, $\lambda_t = \frac{K_\lambda}{t}$

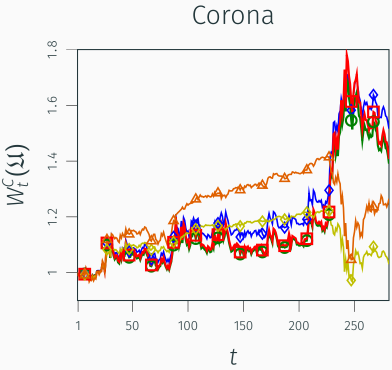
Comparison with State of the Art

- Online Portfolio Optimization:
  - Universal Portfolios (UP) [Cover and Ordentlich (1996)]
  - Online Newton Step (ONS) [Agarwal et al. (2006)]
- Online Portfolio Optimization with Transaction Costs:
  - Online Lazy Updates (OLU) [Das et al. (2013)]

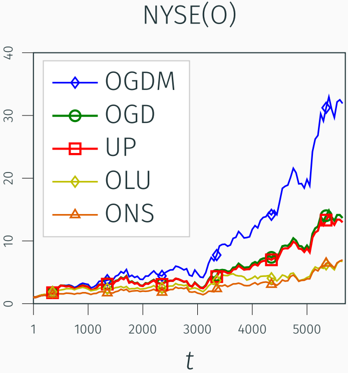
	OGDM	UP	OLU	ONS
R_T	$\mathcal{O}(\sqrt{T})$	$\mathcal{O}(\log T)$	$\mathcal{O}(\sqrt{T})$	$\mathcal{O}(\log T)$
R_T^C	$\mathcal{O}(\sqrt{T})$	-	$\mathcal{O}(T)$	-
Complexity	$\Theta(M)$	$\Theta(T^M)$	$\Theta(M)$	$\Theta(M^2)$

Experiments

Experiments: Wealth $W_T^C(\mathfrak{U})$

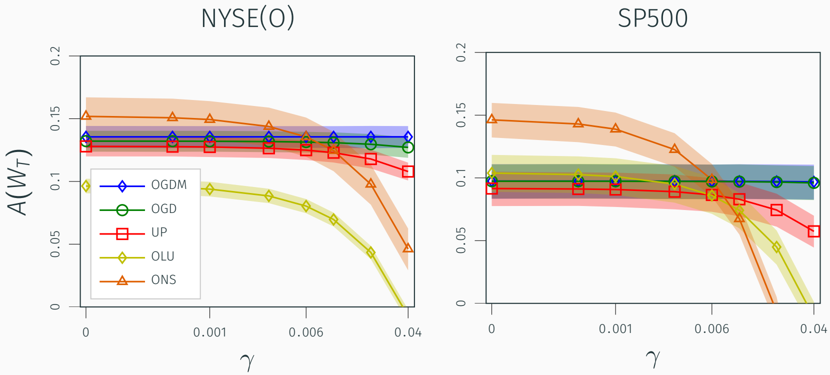


Specific run, on the Corona dataset for $\gamma = 0$



Specific run on 5 stocks of the NYSE(O) for $\gamma = 0.01$

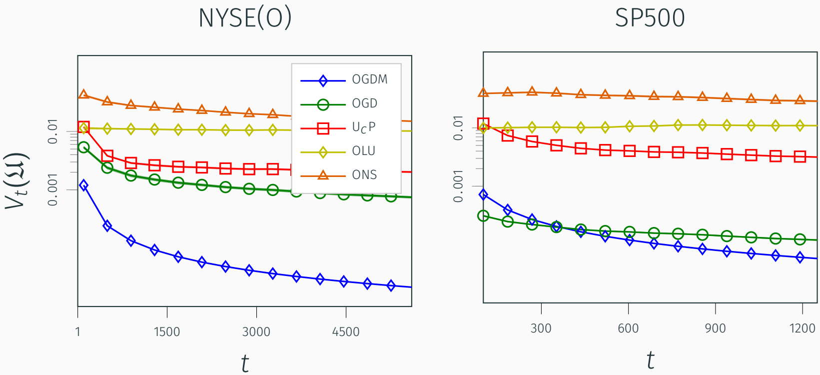
Experiments: Average APY



Average Average Annual Percentage Yield $A(W_T)$ computed on the wealth $W_T^C(\mathbf{x}_{1:T}, \mathbf{r}_{1:T})$:

$$A(W_T) = W_T^{250/T} - 1$$

Experiments: Average variation of the portfolio



Average variation of the portfolio incurred on a varying time horizon t:

$$V_t(\mathfrak{U}) = \frac{C_t(\mathfrak{U})}{\gamma t}$$

Conclusions

Conclusions and Future Work

Contributions:

- OGDM in online portfolio optimization
- Analysis of the total regret of OGDM
- Experimental campaign on real data

Future Works:

- Model stochastic non stationary markets
- Generalize the total regret analysis to other online learning algorithms
- Test on high frequency data
- Make transaction costs more realistic

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Appendix

Experimental Setting

Datasets				
Name	Market	Year Span	Days	Assets
NYSE(O)	New York Stock Exchange	1962 - 1984	5651	36
SP500	Standard Poor's 500	1998 - 2003	1276	25
Corona	Global	2019 - 2020	280	4

Corona Dataset (03/29/2019 - 05/08/2020)		
Ticker	Description	Market Category
SPY	SPDR S&P 500 ETF Trust	Equity
BNDX	Vanguard Bond Index Fund ETF	Fixed Income
DAX	Global X DAX Germany ETF	Equity
VIX	CBOE Volatility Index	Derivatives

State of the Art Algorithms

- Online Portfolio Optimization:
  - Universal Portfolios (UP) [Cover and Ordentlich (1996)]
  - Online Newton Step (ONS) [Agarwal et al. (2006)]
- Online Portfolio Optimization with Transaction Costs:
  - Online Lazy Updates (OLU) [Das et al. (2013)]
  - Universal portfolios with costs
- Heuristics:
  - Passive Aggressive Mean Reversion (PAMR) [Li et al. (2012)]
  - Online Moving Average Reversion (OLMAR) [Li et al. (2015)]