Option Hedging with Risk Averse Reinforcement Learning

Edoardo Vittori

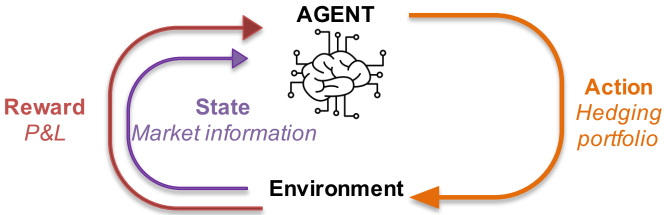
Based on a work done with Michele Trapletti and Marcello Restelli

Outline

- Option Hedging
- Reinforcement Learning Intro
- State of the Art
- Risk Averse RL
- Experimental results
- Conclusions

Option hedging: trading the *underlying asset* in order to minimize the price swings generated by the option (controlling risk).

Reinforcement Learning Intro



- the action $a_t \in [0,1]$ the hedging portfolio
- the state $s_t = (S_t, C_t, \frac{\partial C_t}{\partial S_t}, a_{t-1})$
- the reward $R(s_t, a_t) = C_{t+1}(S_{t+1}) - C_t(S_t) - a_t \cdot (S_{t+1} - S_t) - c(n)$
- transaction costs $c(n) = 0.05 \cdot (|n| + 0.01n^2)$, $n = a_t - a_{t-1}$

Reinforcement Learning Intro

- Returns

$$G(\tau) = \sum_{t=0}^{\infty} \gamma^t R_t$$

- Action-Value function

$$Q_\pi(s, a) = \mathop{\mathbb{E}}_{\tau\sim\pi}[G(\tau)|s_0 = s, a_0 = a]$$

- Objective

$$J = \max_{\pi} \mathop{\mathbb{E}}_{\tau\sim\pi}[G(\tau))]$$

- Policy Search vs Value Based approaches

State of the Art

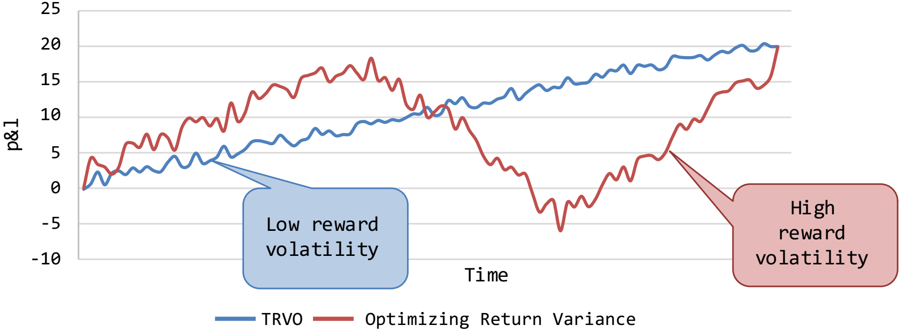
Reinforcement Learning in Finance

- RL in Hedging
  - (Halperin, 2017)
  - (Halperin, 2019)
  - (Kolm and Ritter, 2019a)
  - (Kolm and Ritter, 2019b)
  - (Buehler et al., 2019)
  - (Cao et al., 2019)

Risk Averse Reinforcement Learning

- Reward volatility
  - (Bisi et al., 2020)
- Utility based
  - (Moldovan and Abbeel, 2012)
  - (Shen et al., 2014)
- Coherent Risk Measures
  - (Morimura et al., 2010)
  - (Tamar et al., 2017)
  - (Chow et al., 2017)
- Variance of the returns
  - (Sobel, 1982)
  - (Tamar and Mannor, 2013)
  - (Prashanth and Ghavamzadeh, 2014)

Variance/Volatility relation



$$\sigma_\pi^2 \leqslant \frac{\nu_\pi^2}{(1-\gamma)^2}$$

Reward Volatility

- Reward volatility

$$\nu_\pi^2 = (1-\gamma) \mathop{\mathbb{E}}_{\substack{s_0 \sim \mu \\ a_t \sim \pi(\cdot|s_t) \\ s_{t+1} \sim \mathcal{P}(\cdot|s_t, a_t)}} \left[\sum_{t=0}^{\infty} \gamma^t \left(\mathcal{R}(s_t, a_t) - J_\pi \right)^2 \right]$$

- Mean-volatility objective

$$\eta_\pi := J_\pi - \lambda \nu_\pi^2$$

- Trust Region Volatility Optimization-TRVO (Bisi et al., 2020)

Financial environment

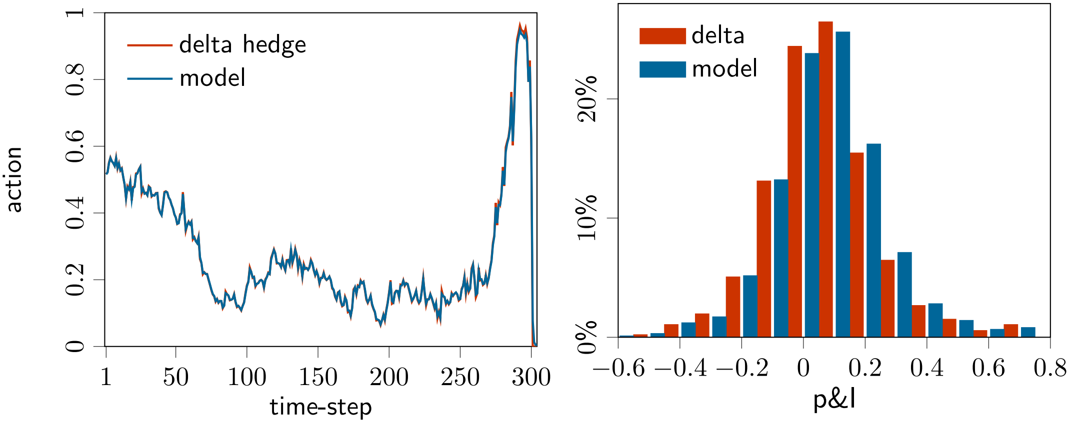
Vanilla call option

- time to maturity = 60 days
- unitary notional
- implied volatility = 20%
- interest rates = 0
- $K (= S_0) = 100$
- starting price (ATM) option ~ 3.24
- starting delta $= 0.5$

Simulated Market

- geometric brownian motion $dS_t = \mu S_t dt + \sigma S_t dW_t$
- no drift
- $\sigma = 20\%$
- $S_0 = 100$
- 5 time steps per day

Experimental Results: no Transaction Costs

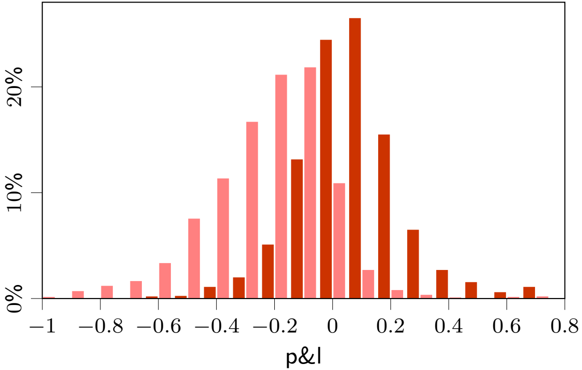


⇒ delta hedge with no costs → average p&l $\sim$ 0, volatility $\sim$ 0.16

Adding costs

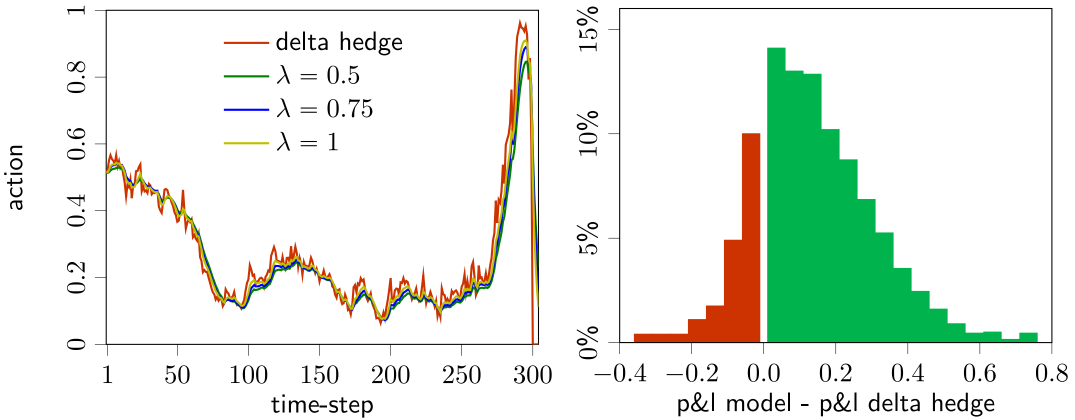
We considered h.c.$\sim 0.05|a|$, $\sim$ the Euro Stoxx 50 or FTSE MIB future.

- More liquid listed products (S&P 500) have lower minimal costs
- Less liquid listed or OTC (vanilla, flow) instruments have significantly higher costs



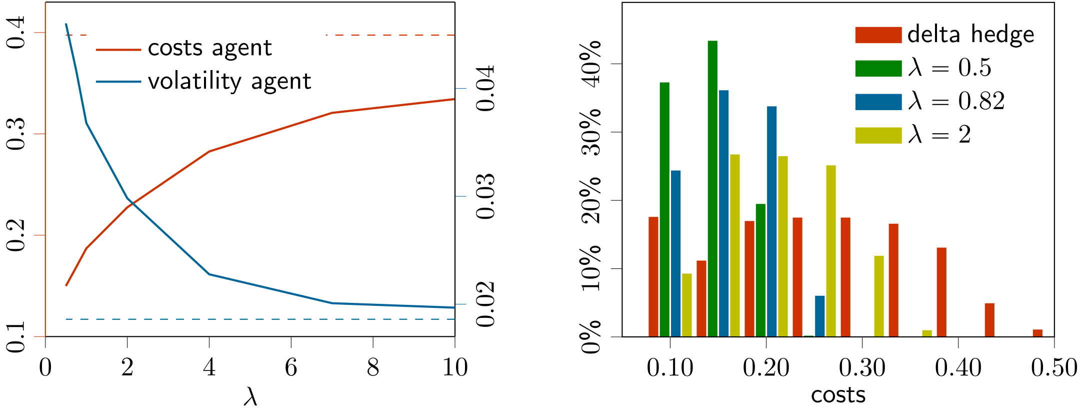
- The agent now has something to optimize: costs vs. volatility
- Costs give a role to the risk aversion factor

Experimental Results: with Transaction Costs

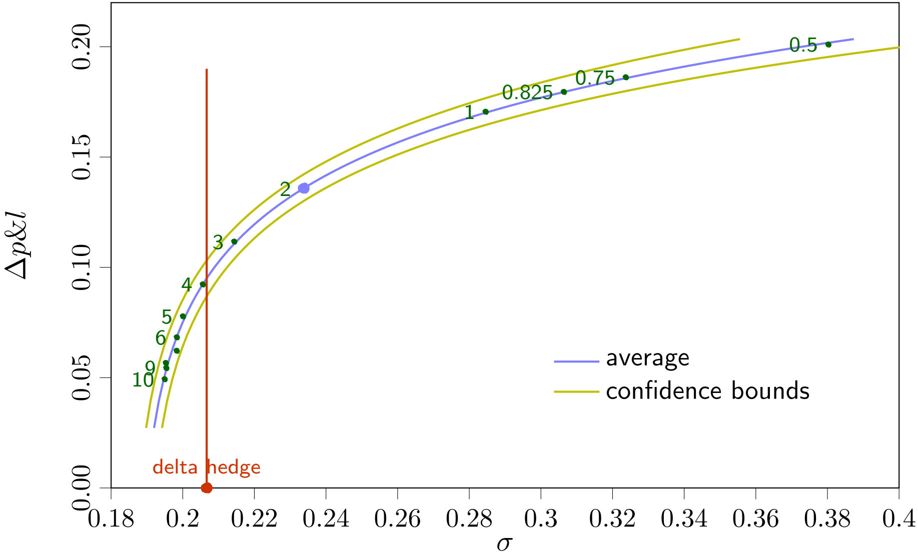


⇒ delta hedge with no costs → average p&l ∼ -0.3, volatility ∼ 0.18

Experimental Results: Cost Analysis



Experimental Results: Pareto Plot



Conclusions

Contributions

- Proved experimentally that the hedging strategy learnt by the model dominates the delta hedge

Future works

- Extend to more complex derivatives
- Extend to a portfolio of options
- Decide not only how much but also when to hedge

Contacts

- Edoardo Vittori - edoardo.vittori@polimi.it
- Michele Trapletti - michele.trapletti@intesasanpaolo.com
- Marcello Restelli - marcello.restelli@polimi.it

Thank You for Your Attention!

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