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Dark-Pool Smart Order Routing: a Combinatorial Multi-armed Bandit Approach

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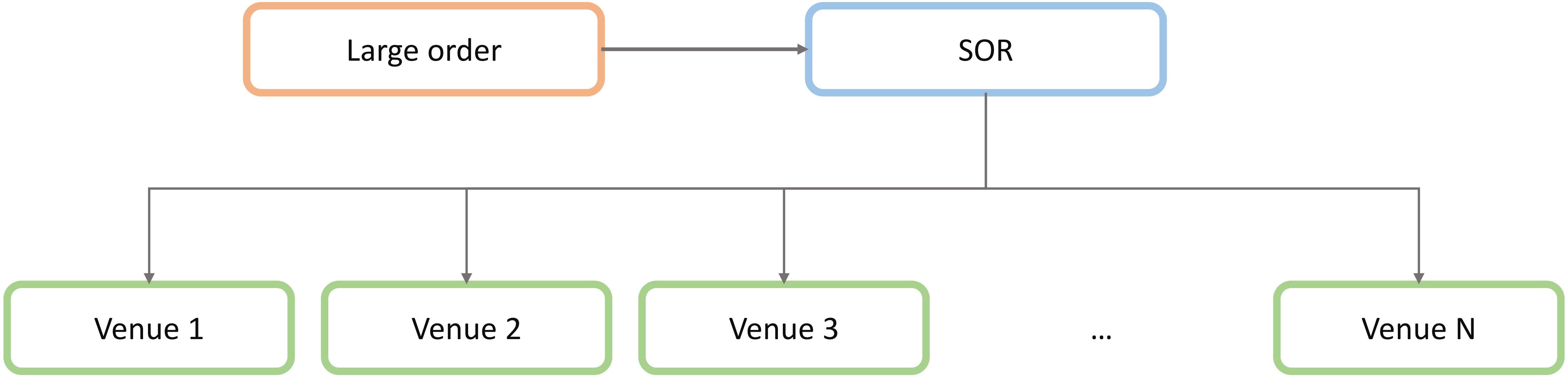
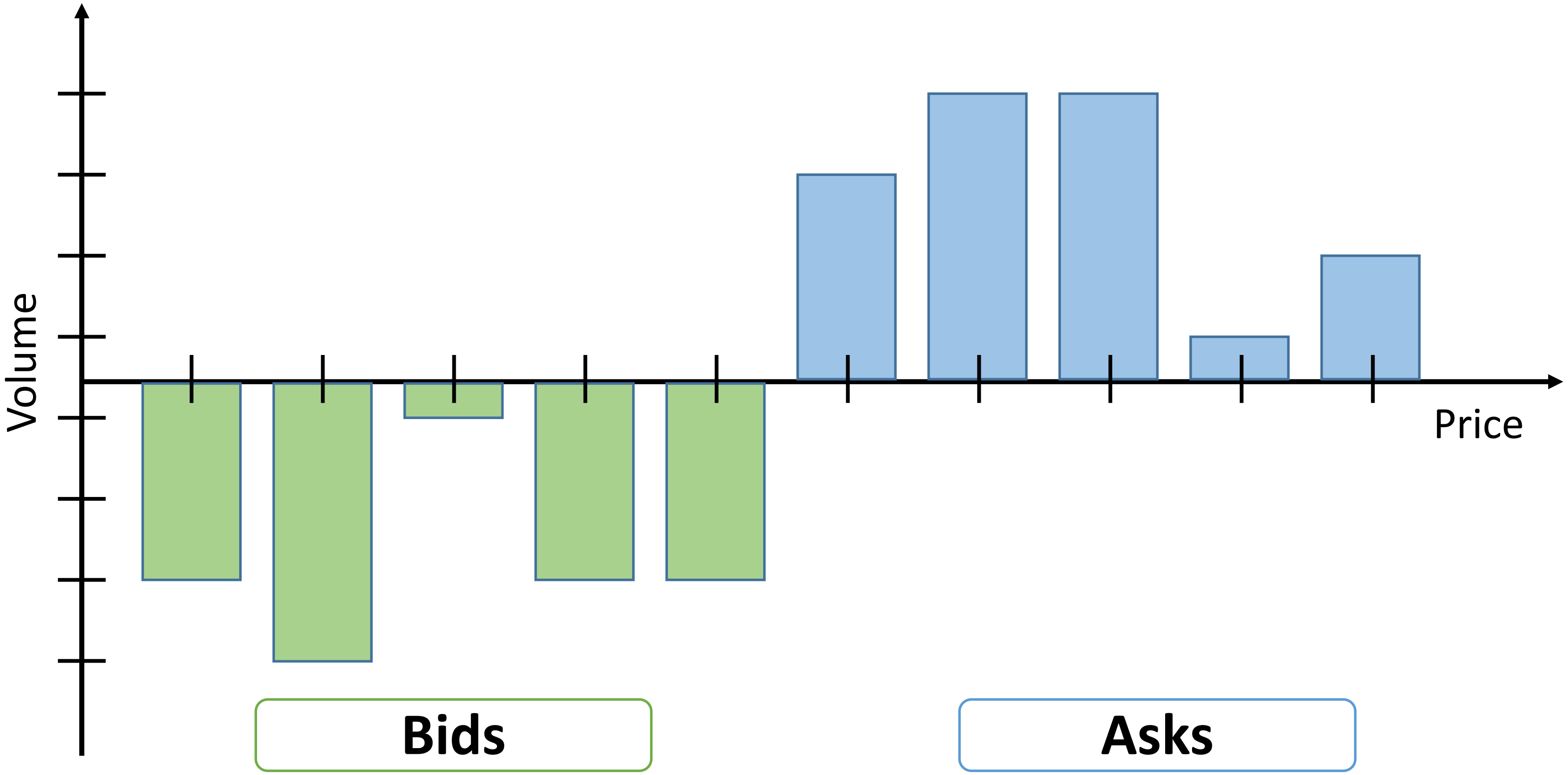
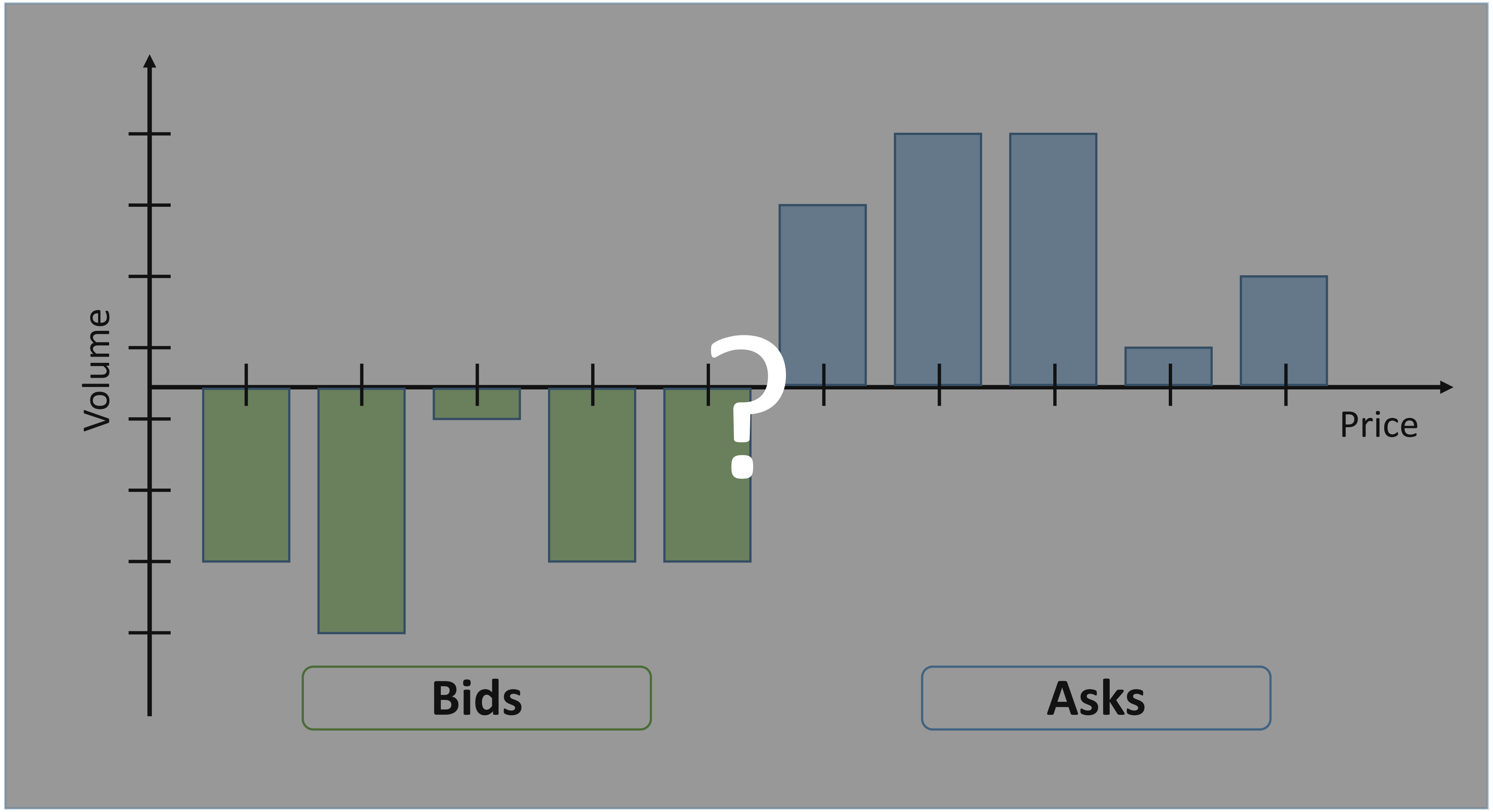
Smart **O**rder **R**outing (SOR): optimally splitting an order over multiple venues



Regulated Exchanges - Limit Order Book




Volume
?
Price
Bids
Asks

Dark Pool Smart Order Routing - DPSOR

Task

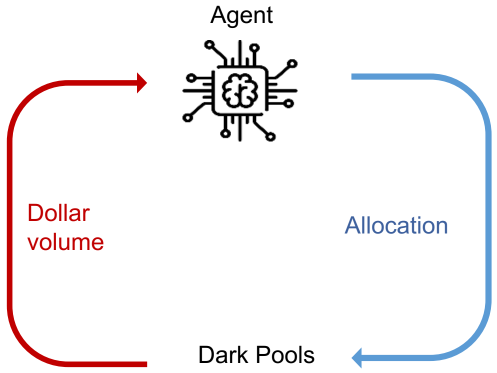
- Create and maintain a **estimate** of **hidden liquidity** of multiple dark pools
- Make optimal joint **routing and pricing decisions**
- Optimize the **dollar volume**

Assumptions

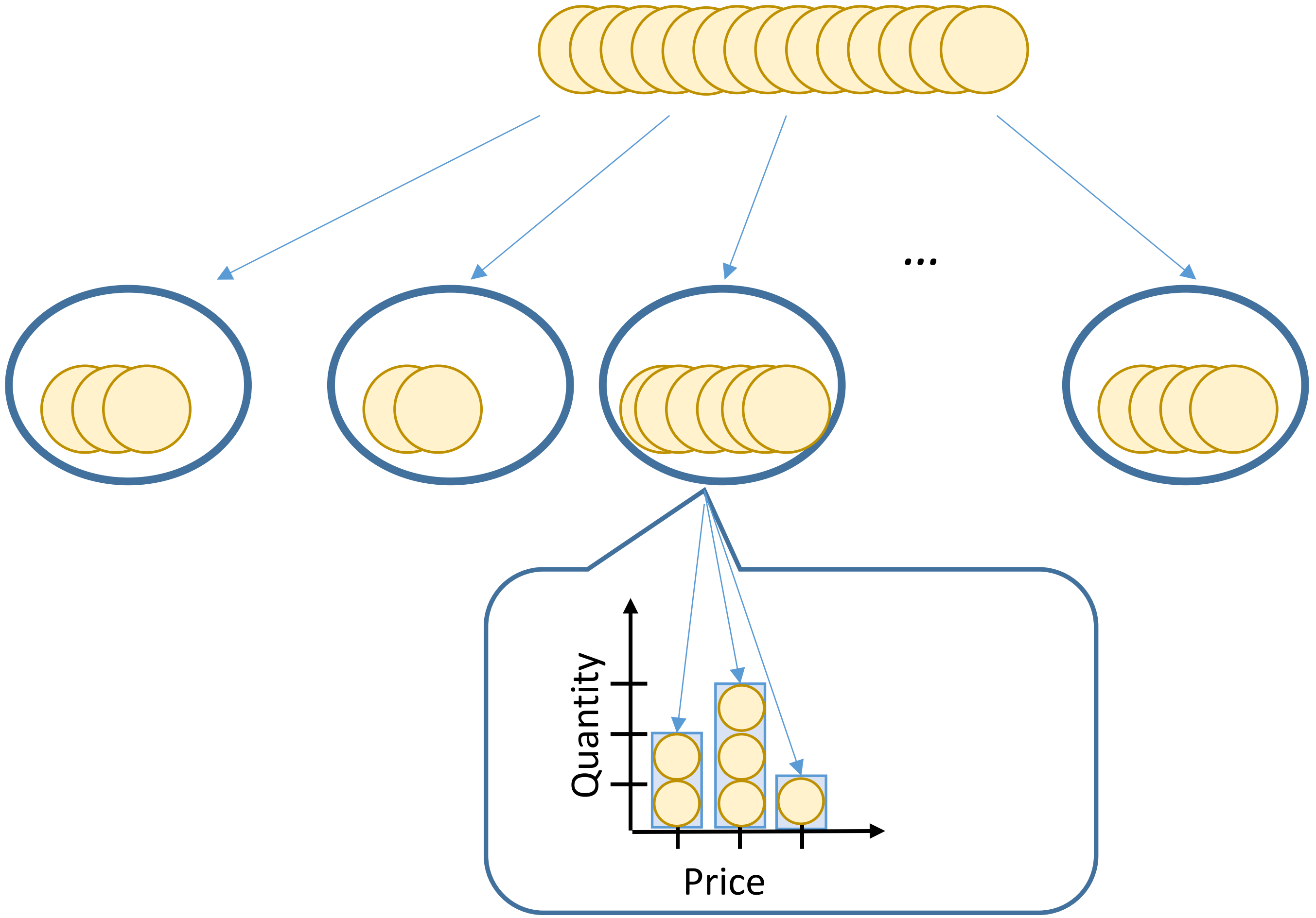
- **Multiple dark pools** for a single asset
- Stationary liquidity
- **Limit orders** are admitted

Formulation

- **Sequential decision problem** where at each round *t*, an agent, given a volume V of shares to execute, must maximize the dollar volume by allocating the shares across K dark pools, specifying the price



Joint routing and pricing allocation

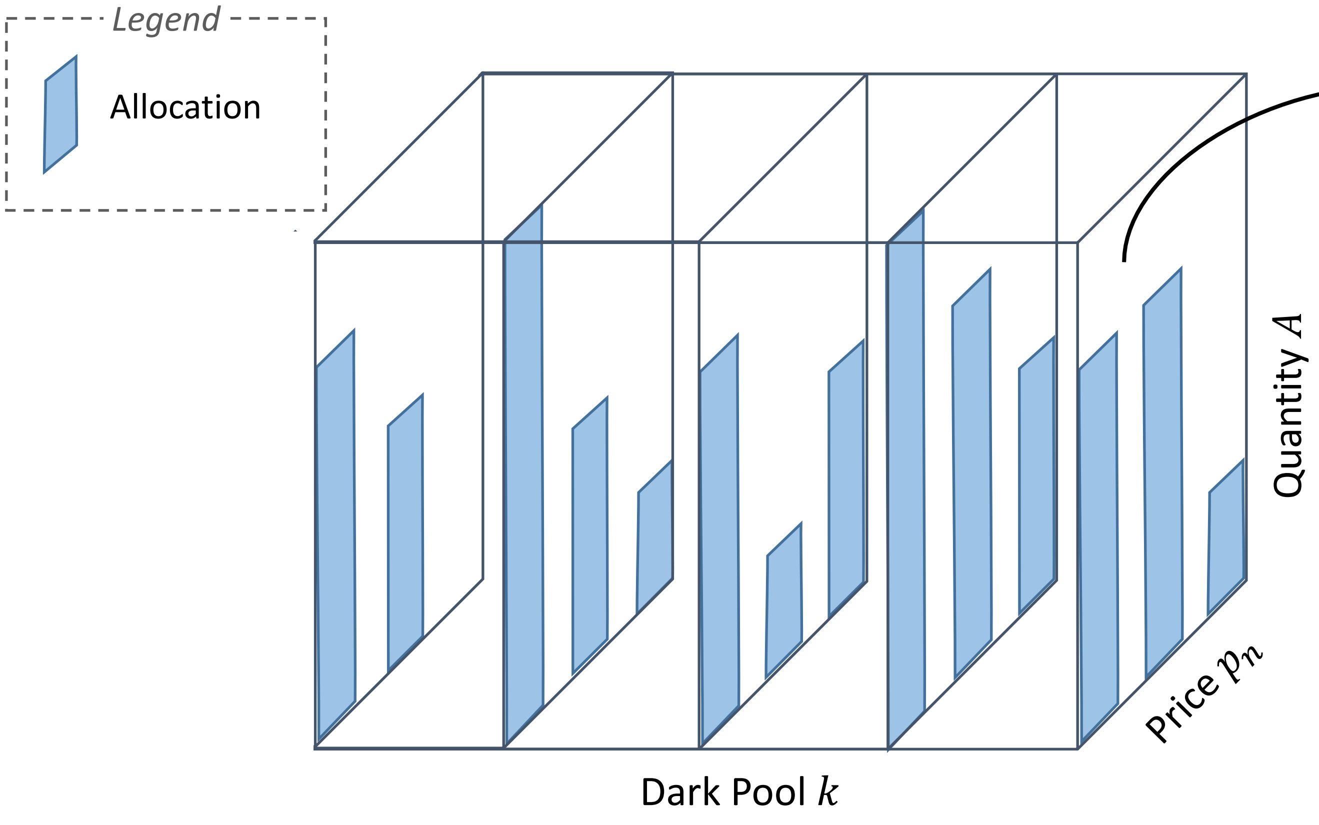


V units to sell

Allocate to K dark pools

Specify amount to allocate at a specific price

Problem formalization and notation



A_{kn}^{t}: amount allocated at round t to dark pool k at price p_n

- We have the constraint that

$$\sum_{k=1}^{K}\sum_{n=1}^{N} A_{kn}^{t} = V$$

- Our objective is the allocation that maximizes dollar volume

$$R_t(\mathfrak{U}) = \sum_{k=1}^{K}\sum_{n=1}^{N} r_{kn}^{t}\, p_n.$$

Censored feedback

$$r_{kn}^{t} = \min\{A_{kn}^{t}, s_{kn}^{t}\}$$

s_{kn}^{t} is the actual liquidity present at time t in dark pool k at price p_n

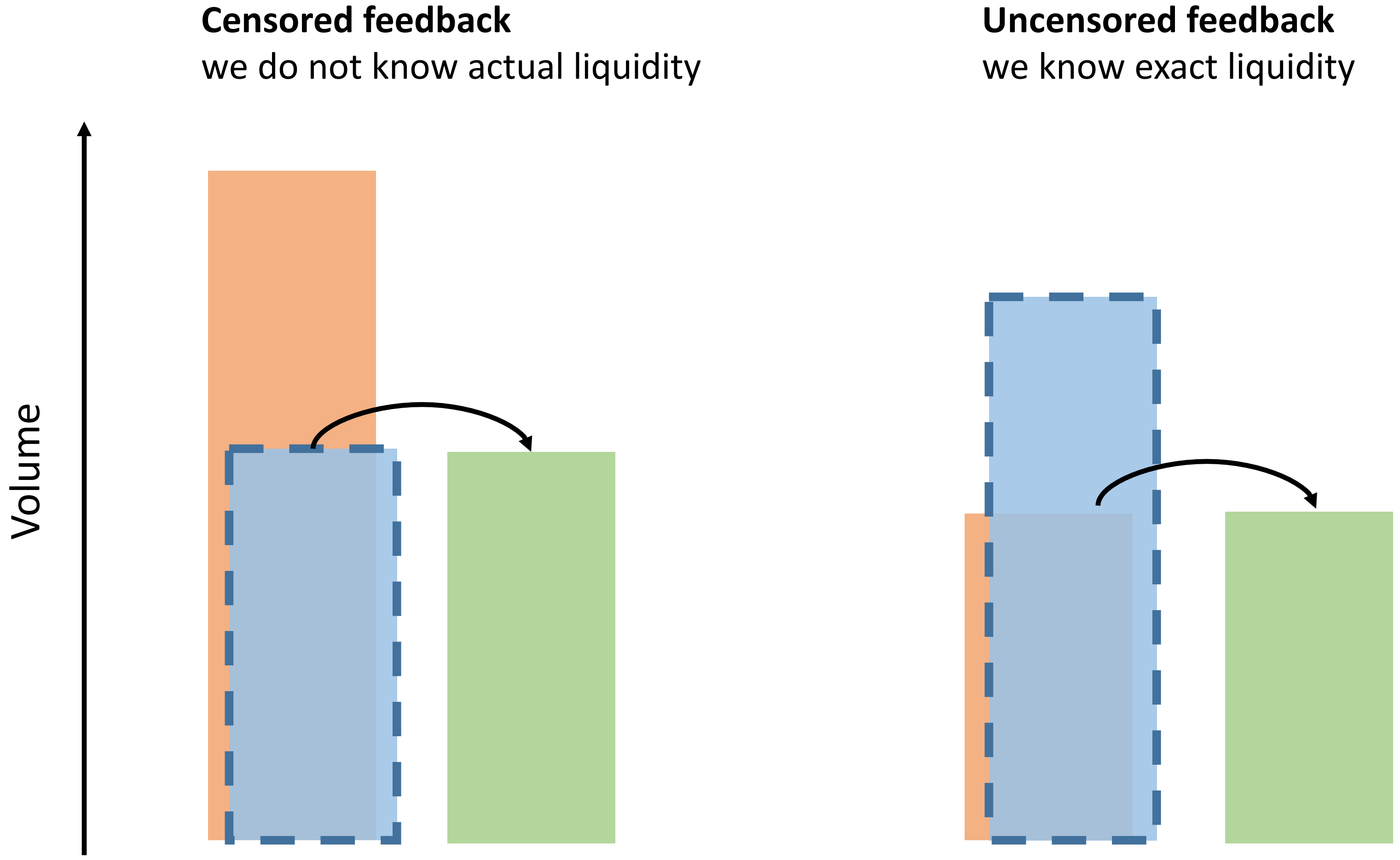
Censored feedback

Legend

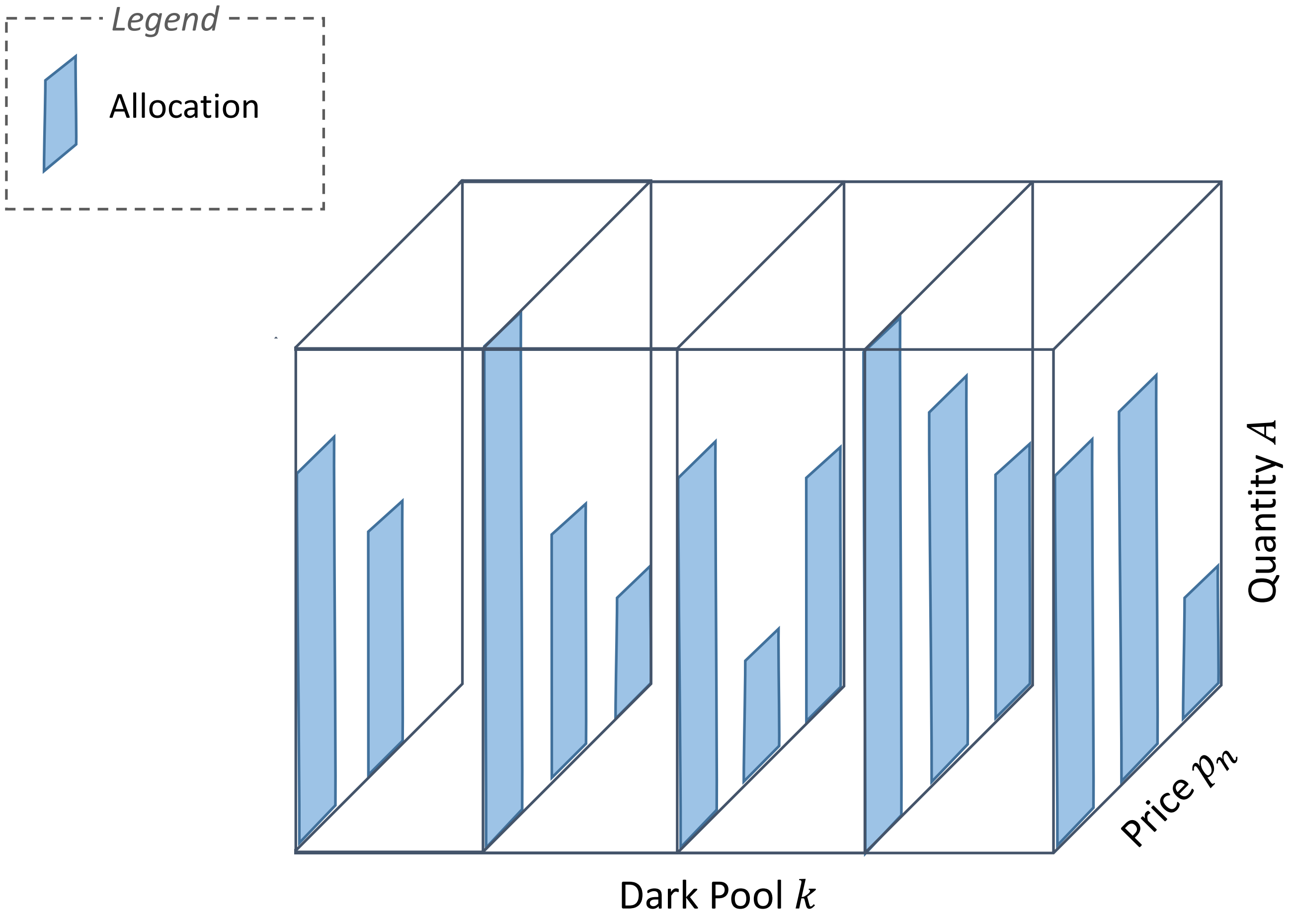
Actual liquidity s_t

Allocation A_t

Executed order



Combinatorial MAB [Chen et al., 2013]



- We are in a CMAB setting, where the superarms are all the combinations of A^t_{kn} which satisfy the following constraint:

$$\sum_{k=1}^{K} \sum_{n=1}^{N} A^t_{kn} = V$$

- We want to minimize pseudo-regret w.r.t. the expected dollar value of the optimal superarm r^*

$$Reg_t(\mathfrak{U}) := t\, r^* - \sum_{h=1}^{t} \sum_{k=1}^{K} \sum_{n=1}^{N} \mathbb{E}[r^h_{kn}] \mathbb{1}\{A^h_{nk} > 0\}\, p_n$$

$$r^t_{kn} = \min\{A^t_{kn}, s^t_{kn}\}$$

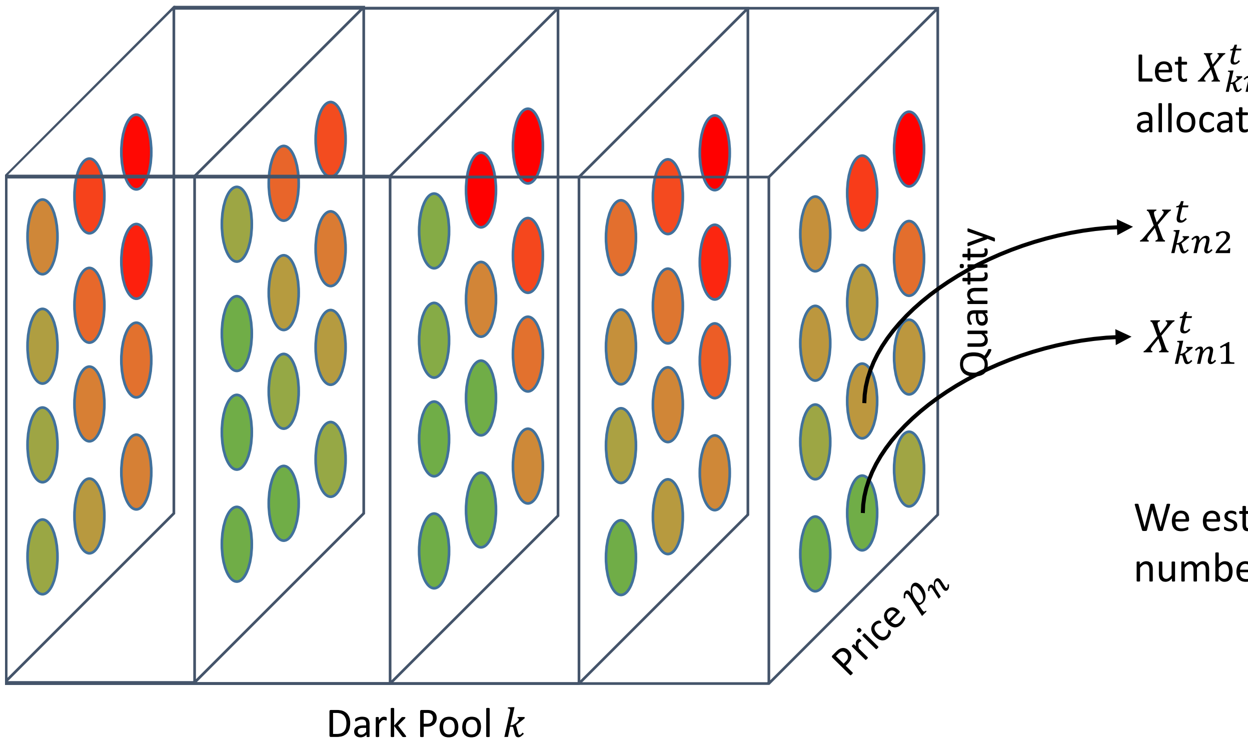
Estimating liquidity

Legend

Triplet dark pool, price, quantity

Expected probability of success

High *Low*



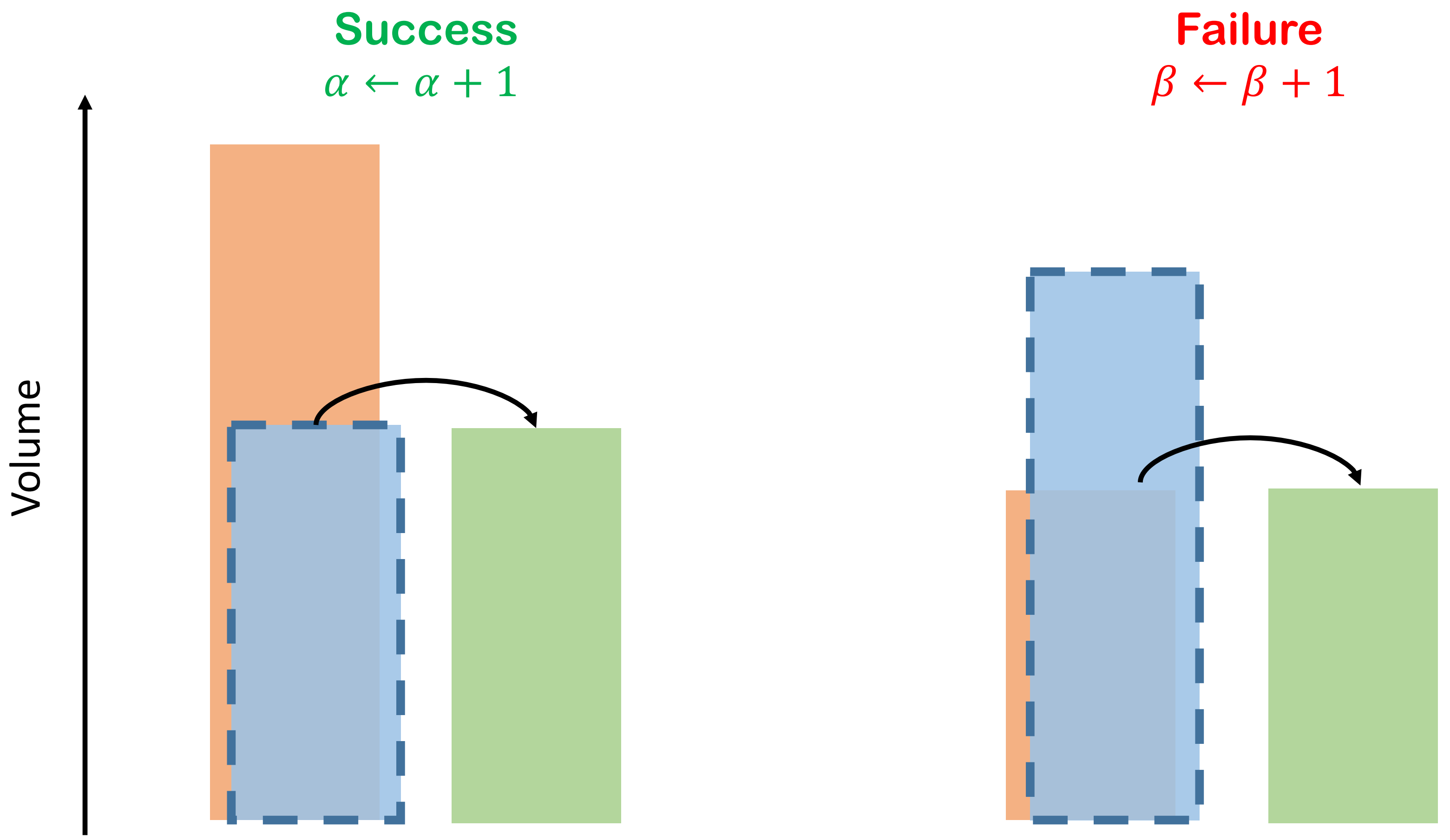
Let X_{knv}^{t} the probability that a specific allocation is successful

We estimate X_{knv}^{t} by counting the number of successes and failures

Counting successes α and failures β

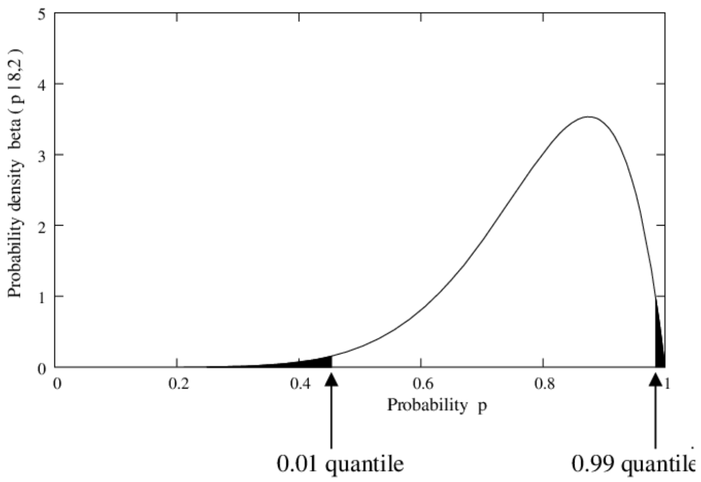
Legend

- Actual liquidity s_t
- Allocation A_t
- Executed order



DP-CMAB Algorithm – θ Selection

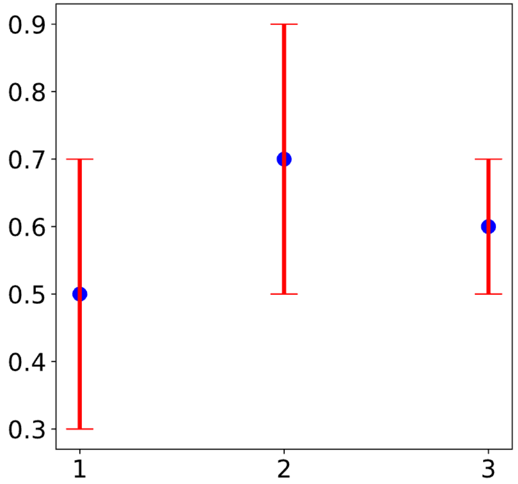
DP-BAYES-UCB



Compute the quantiles of the Beta distribution

$$\theta_{knv}^{t} = v\,\underbrace{Q\left(1 - \frac{1}{t(\log T)^5}; Beta(\alpha_{knv}^{t}, \beta_{knv}^{t})\right)}_{X_{knv}^{t}}$$

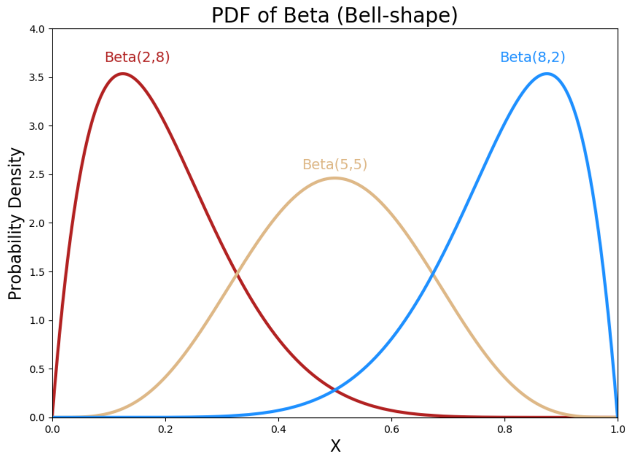
DP-CUCB



Mean and uncertainty

$$\theta_{knv}^{t} = v\,\underbrace{\left(\frac{\alpha_{knv}^{t} - 1}{\alpha_{knv}^{t} + \beta_{knv}^{t} - 2} + \sqrt{\frac{2\log(t)}{\alpha_{knv}^{t} + \beta_{knv}^{t} - 2}}\right)}_{X_{knv}^{t}}$$

DP-TS

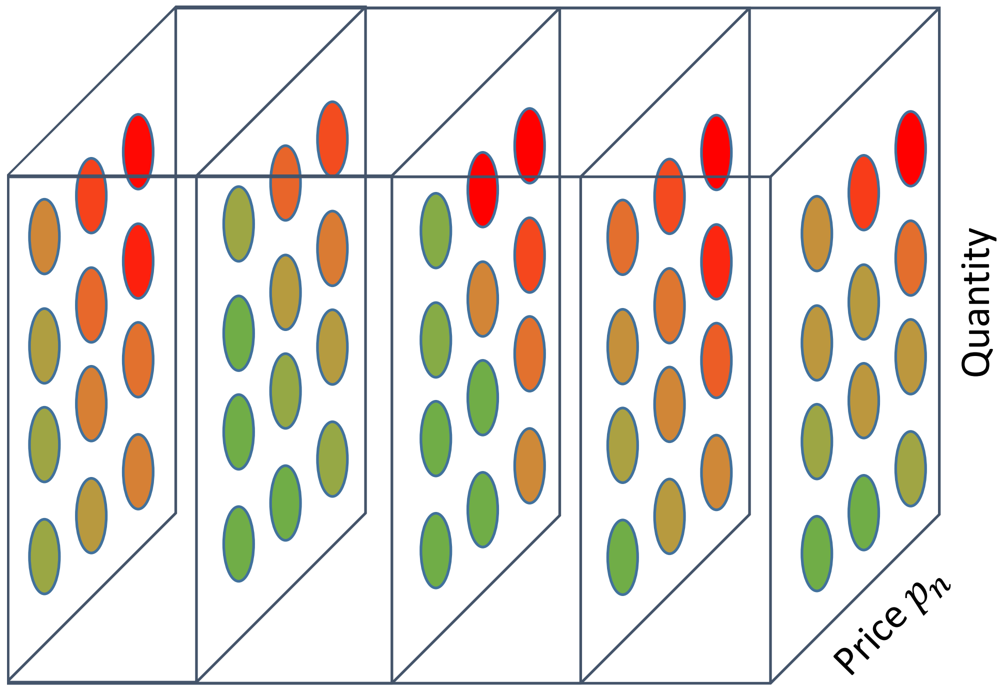


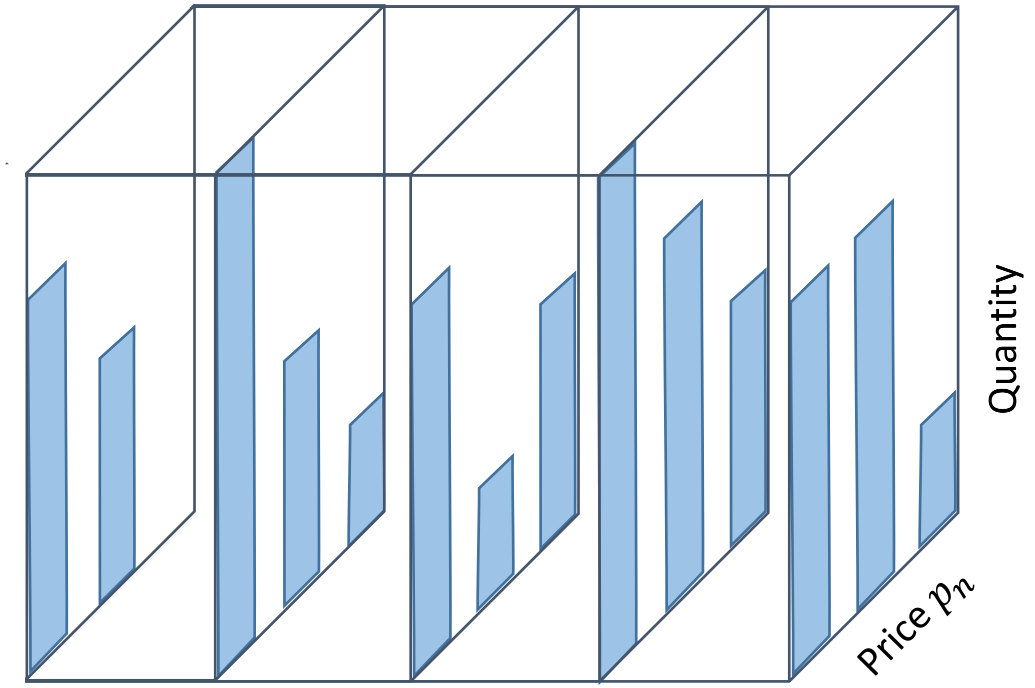
Sample from the Beta distribution

$$\theta_{knv}^{t} \sim v\,\underbrace{Beta\left(\alpha_{knv}^{t}, \beta_{knv}^{t}\right)}_{X_{knv}^{t}}$$

Translating liquidity to allocation

$$\boldsymbol{\theta}_t = v\boldsymbol{X}_t \qquad \text{Opt}(\boldsymbol{\theta}_t) \rightarrow \boldsymbol{A}_t$$



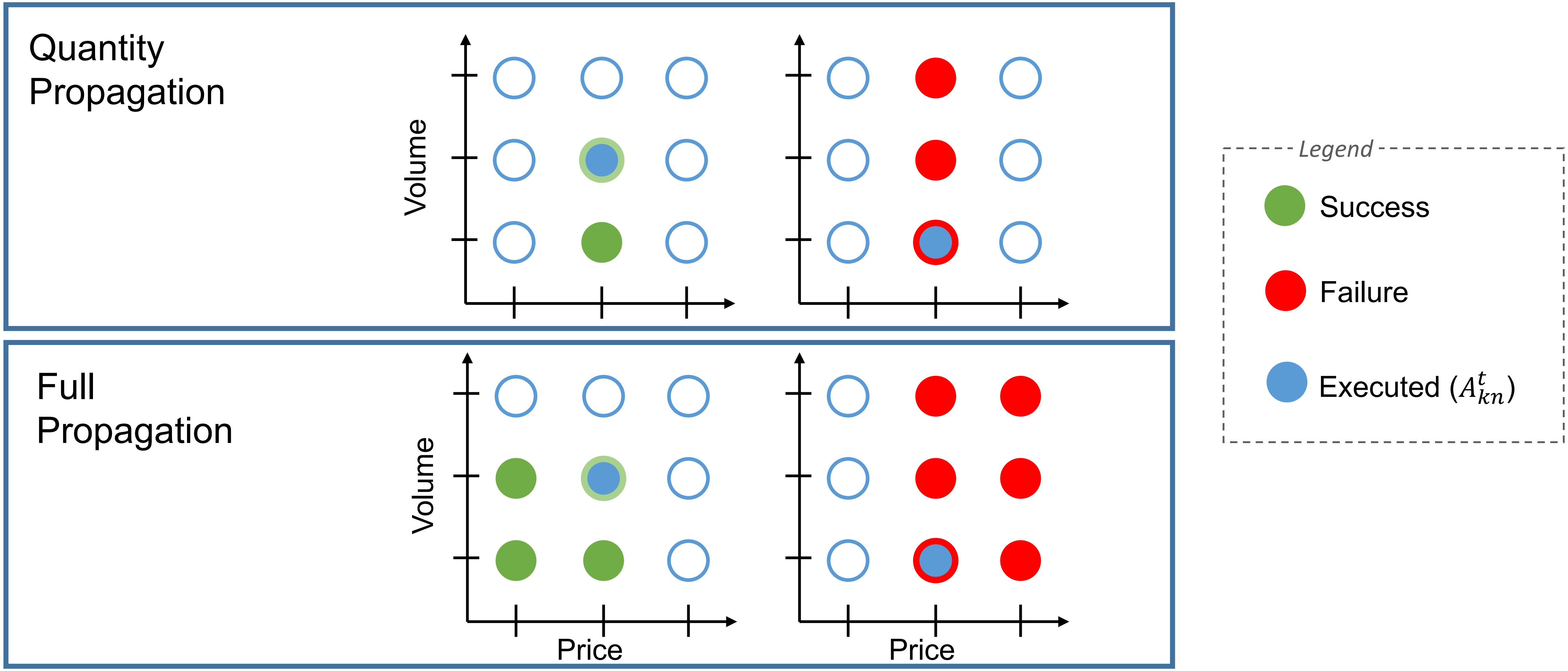


DP CMAB High Level Pseudo Code

At each round t:

- Calculate the liquidity estimate $\boldsymbol{\theta_t}$ using $\boldsymbol{\alpha_t}, \boldsymbol{\beta_t}$ and the appropriate update Bayes, CUCB or TS
- Calculate the action matrix $\boldsymbol{A_t} \leftarrow \mathrm{Opt}(\boldsymbol{\theta_t})$
- Play allocation $\boldsymbol{A_t}$
- Receive feedbacks $\boldsymbol{r_t}$ from played arms
- Calculate the parameters $\boldsymbol{\alpha_{t+1}}$ and $\boldsymbol{\beta_{t+1}}$

Can we do better?

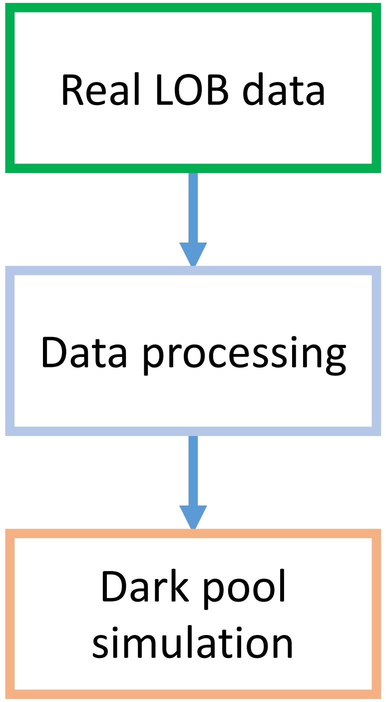


Experimental setup and Liquidity simulation

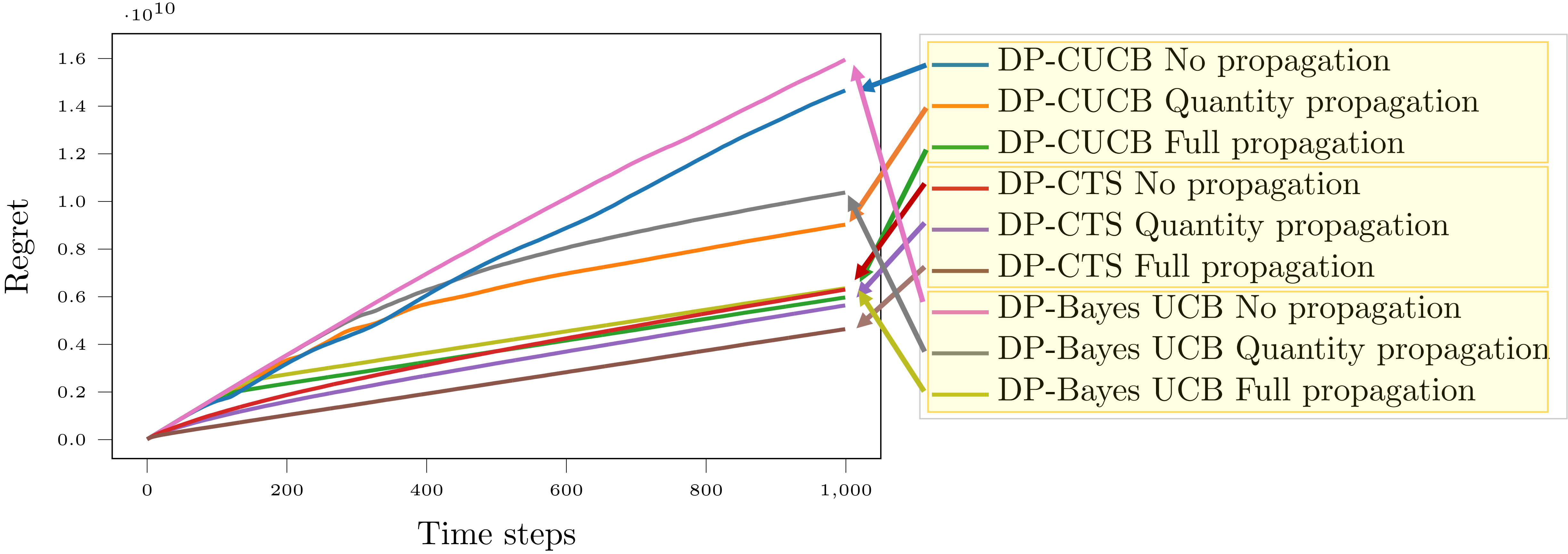
Experimental setup

- Selling V = 10 units
- K = 10 dark pools
- Prices in {90, 91, …, 100}
- Rounds T = 1000
- Results averaged over 20 runs

Liquidity simulation



Experimental results – Different strategies and propagations



Related Works - The Dark Pool problem

Related works

Censored Exploration and the Dark Pool Problem, Ganchev et al. [2009]

- Finds an optimal allocation strategy based on the Kaplan-Meier estimator

Optimal Allocation Strategies for the Dark Pool Problem, Agarwal [2010]

- Extends Ganchev et al. [2009] to an adversarial scenario

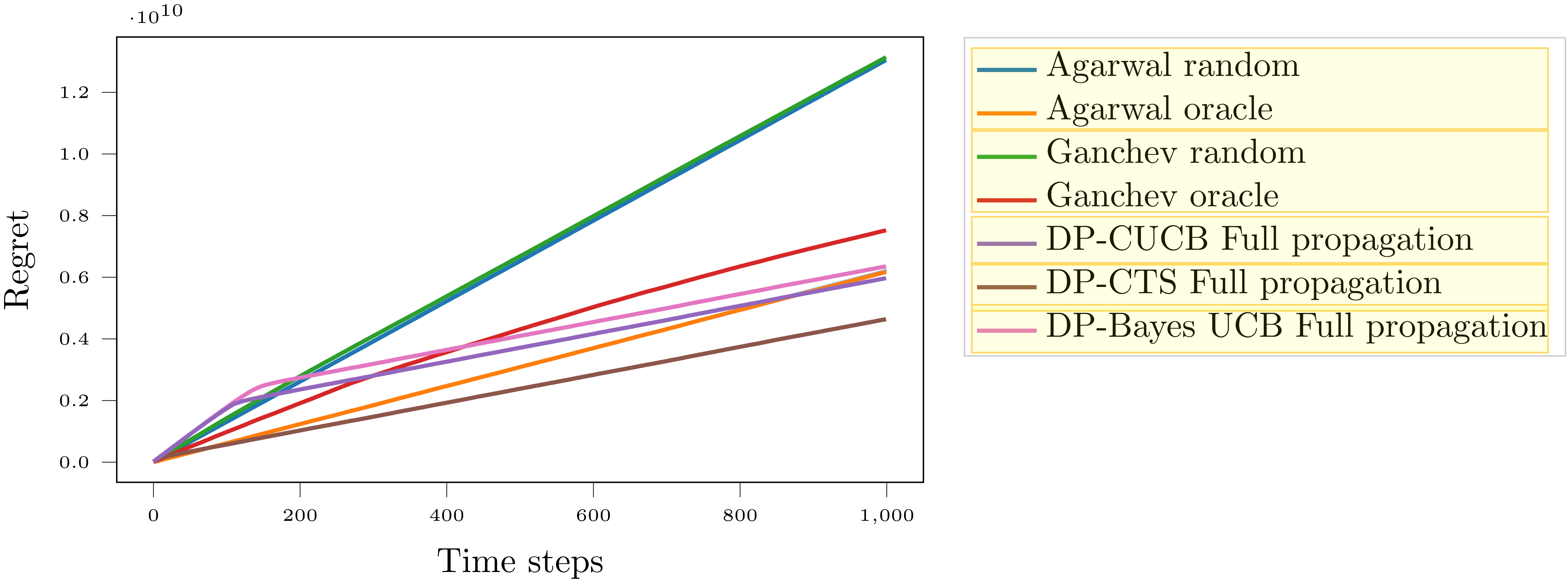
Limitations

They do not allow the agent to specify the transaction price and do not take advantage of domain knowledge

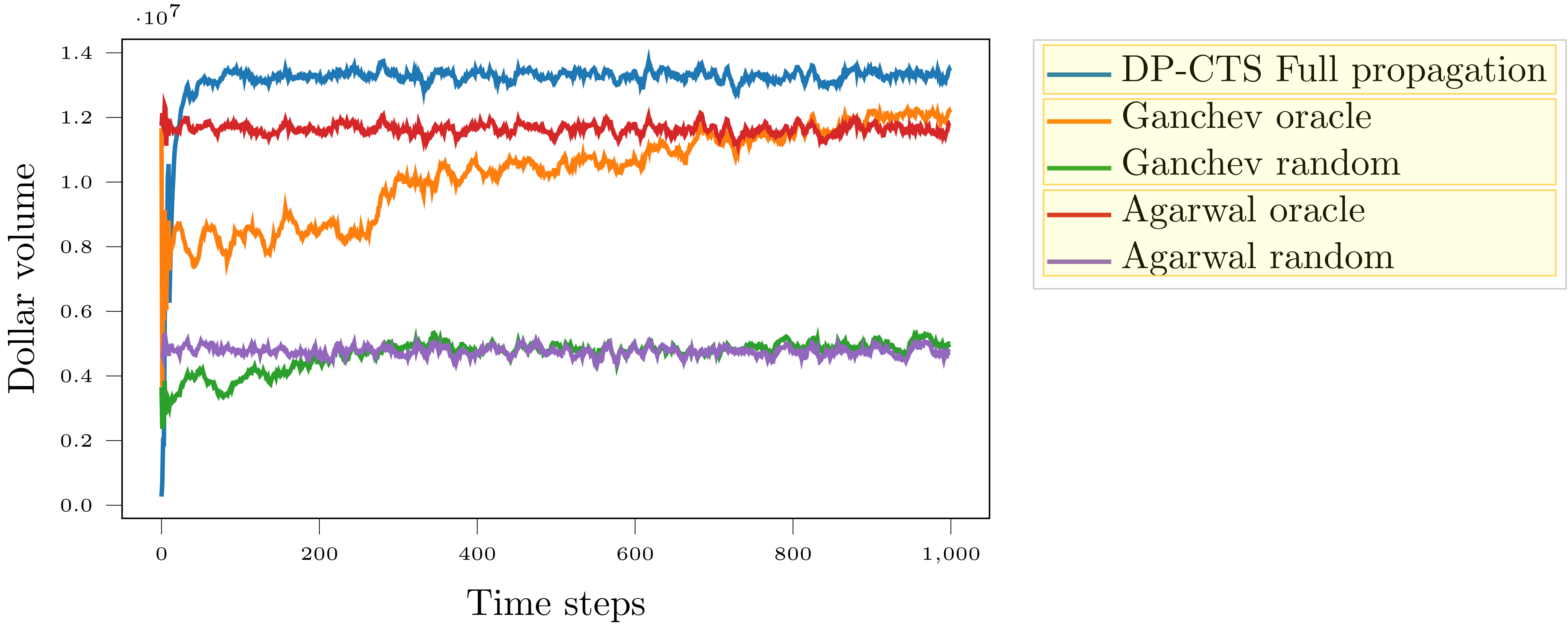
Price selection

- Random: selects a random price at the beginning of each round
- Oracle: selects best single price which maximizes the expected cumulated dollar volume across the run.

Experimental results – Comparison with baselines



Experimental results – Dollar volume



Conclusions

We developed the DP-CMAB algorithm that:

- extends the limitations of state-of-the-art algorithms
- exploits the knowledge about the financial setting to improve its allocation policy
- empirically outperforms state-of-the-art algorithms in a realistic scenario

Further extensions:

- regret proof of propagation updates
- nonstationary (ADVERSARIAL) dark pool liquidity

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